

DIRECTIONS To receive full credit, you must provide complete legible solutions to the following problems in the space provided. Transfer all your answers to the space provided on the test paper

1. In this problem, $y = \frac{1}{x^2 + c}$ is a one-parameter family of solutions of the first-order DE

$$y' + 2xy^2 = 0, \quad y(4) = 1/15$$

- a. Find a solution of the first-order IVP consisting of this differential equation and the given initial condition.
- b. Give the largest interval I over which the solution is defined. (Enter your answer using interval notation.)

2. Verify that $y = \tan(x + c)$ is one parameter family of solutions of the differential equation $y' = f(x, y) = 1 + y^2$

- a. Give the domain of the solution function.
- b. On what part of the xy-plane are both $y' = f(x, y)$ and $f_y(x, y)$ are continuous

- c. Determine the largest interval I_0 of existence and uniqueness for the solution of the initial value problem with IC: $y(0) = 0$

3. Determine a region in the xy plane in which the given differential equation has a unique solution whose graph passes through the point (x_0, y_0) .

$$x \frac{dy}{dx} = y.$$

4. Determine a region in the xy plane for which the given differential equation has a unique solution whose graph passes through a point (x_0, y_0) .

$$(1 + y^3)y' = x^2$$

5. In this problem, $x = c_1 \cos(t) + c_2 \sin(t)$ is a two-parameter family of solutions of the second-order DE $x'' + x = 0$.

Find a solution of the second-order IVP consisting of this differential equation and the given initial conditions. $x(\pi/2) = 0$, $x'(\pi/2) = 1$